

# INTRODUCTORY CALCULUS FOR THE BIOLOGICAL SCIENCES

<b>Questions</b>	110
<b>Field</b>	Mathematics
<b>Target Audience</b>	Biological Sciences
<b>Target Level</b>	First-year Undergraduate
<b>Topics</b>	<ul style="list-style-type: none"><li>▪ Introduction to Functions</li><li>▪ Composite and Inverse Functions</li><li>▪ Trigonometric Functions</li><li>▪ Logarithms and Exponents</li><li>▪ Sequences and Finite Series</li><li>▪ Limits and Continuity</li><li>▪ Derivatives</li><li>▪ Curve Sketching</li><li>▪ Differentials</li><li>▪ Linear Approximation</li><li>▪ Taylor Polynomials</li><li>▪ Difference Equations</li><li>▪ Log-Log Graphs</li><li>▪ Anti-Differentiation</li><li>▪ Definite Integrals</li></ul>

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## Outline

The material in this module is designed to cover a single-semester introductory calculus course for biological sciences students at the first-year university level. The questions are designed to span the topics listed above, allowing for practice, homework or testing throughout the semester.

The vast majority of the questions are algorithmic and take numeric or algebraic responses. Information fields are included on all questions indicating topics. There are questions designed to test the students' mathematical skills and ability to apply these skills to real world problems.

Sample homework assignments are provided, spanning the course material.

The included Maple TA Syntax Sheet can be distributed to the students as it covers many common response scenarios.

# Maple TA Syntax Sheet

Expression	Entry Syntax
$x \cdot y$	<code>x*y</code>
$\frac{x}{y}$	<code>x/y</code>
$x^y$	<code>x^y</code>
$\frac{a}{b \cdot c}$	<code>a/(b*c)</code> (although it will accept <code>a/b/c</code> )
$\sqrt{x}$	<code>sqrt(x)</code> or <code>x^(1/2)</code> (do not use <code>x^0.5</code> )
$x^{\frac{2}{3}} = \sqrt[3]{x}$	<code>x^(2/3)</code>
$ x $	<code>abs(x)</code>
$\ln(x)$	<code>ln(x)</code>
$\log_n(x)$	<code>log[n](x)</code>
$e^x$	<code>exp(x)</code>
$e$	<code>e</code> or <code>exp(1)</code>
$\pi$	<code>pi</code> or <code>Pi</code>
$\infty$	<code>infinity</code>
$\sin^2(x) = (\sin(x))^2$	<code>sin(x)^2</code> or <code>(sin(x))^2</code>

## Notes

Maple TA likes to make the following substitutions when displaying equations

Simple Form	Maple TA Will Show
$\sec(x)^2$	<code>1+tan(x)^2</code>
$\csc(x)^2$	<code>1+cot(x)^2</code>
$\sec(x)*\tan(x)$	<code>sin(x)/cos(x)^2</code>
$\csc(x)*\cot(x)$	<code>cos(x)/sin(x)^2</code>

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# **Introductory Calculus for the Biological Sciences**

## **Sample Tests**

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**Summer 2012**

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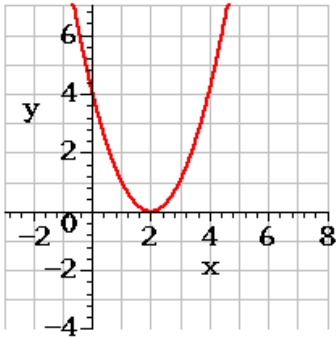
**Test 13: Definite Integrals**

## Test 1: Introduction to Functions

### Question 1: Score 1/1

#### Your response

Determine the function, its natural domain, and range from its given graph.  
(for  $-\infty$  or  $\infty$  use -infinity and infinity respectively).



Correct

Hint: The function is based on shifts and/or reflections of the function  $f(x) = x^2$

$f(x) = (x-2)^2$  (33%)

For the domain & range, enter your answer in interval notation. For example if  $0 \leq x < \infty$ , enter  $[0, \infty)$ .

What is the domain of the function?

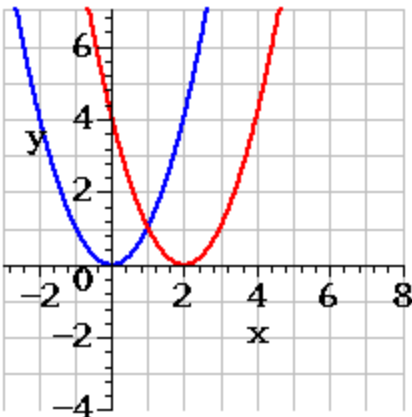
$(-\infty, \infty)$  (33%)

What is the range of the function?

$[0, \infty)$  (33%)

#### Comment:

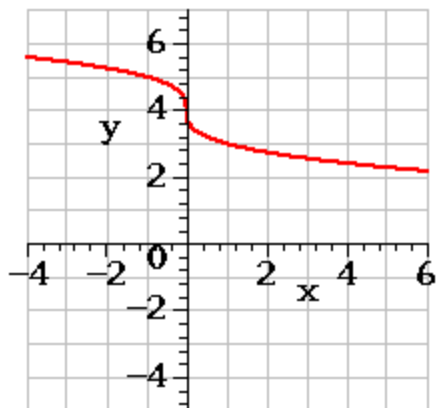
Here is the original graph,  $y = x^2$  with the function  $y = (x - 2)^2$ .



## Question 2: Score 1/1

Your response

Which one of the following equations represents the sketched graph?

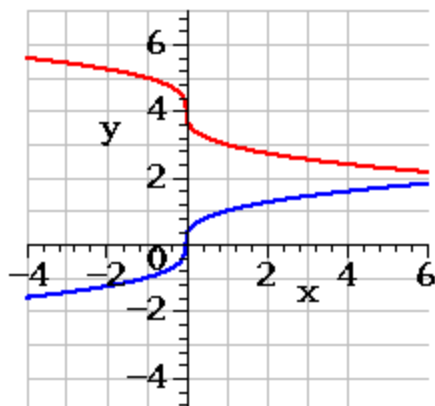


Correct

$-x^{1/3} + 4$  (100%)

Comment:

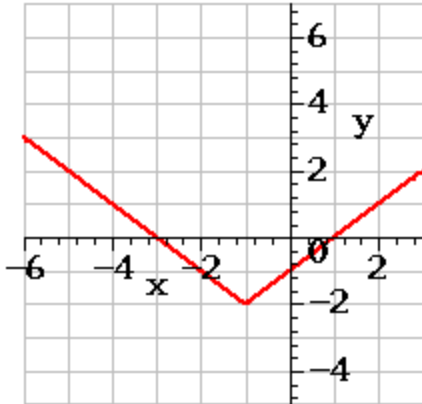
Here is the graph of the base function  $y = x^{1/3}$ , with the function  $\text{surd}(-x, 3) + 4$ .



### Question 3: Score 1/1

#### Your response

Determine the function, its natural domain, and range from its given graph. (for  $-\infty$  or  $\infty$  use -infinity and infinity respectively).



Correct

Hint: The function is based on shifts and/or reflections of the function  $f(x) = |x|$ .

Enter `abs(x+1)` for  $|x + 1|$ .

$f(x) =$  `abs(x+1)-2` (33%)

For the domain and range, enter your answer in interval notation. For example if  $0 \leq x < \infty$ , enter  $[0, \text{infinity})$ .

What is the domain of the function?

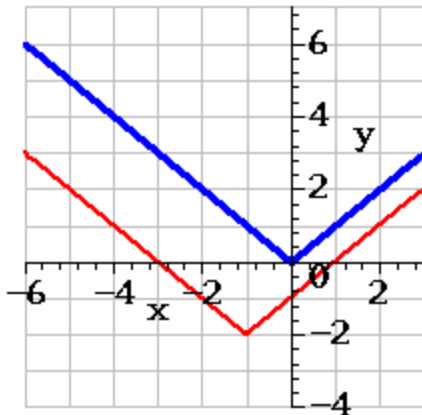
`(-infinity, infinity)` (33%)

What is the range of the function?

`[-2, infinity)` (33%)

Comment:

Here is the original graph,  $y = x^2$  with the function  $\text{abs}(x+1)-2$ .



#### Question 4: Score 1/1

Your response

The natural domain of  $f(x) = \frac{\sqrt{25 - x^2}}{\lfloor x \rfloor}$  is

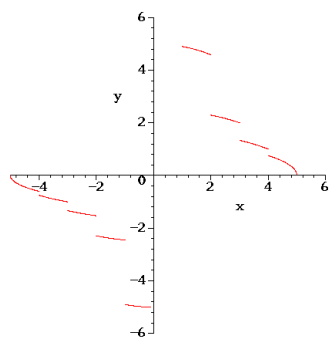
$[-5, 0) \cup [1, 5]$  (100%)



Correct

Comment:

Note the difference between  $(a, b)$  and  $[a, b)$ . The graph looks like this!



#### Question 5: Score 1/1

Your response

Assume that the growth rate of a plant,  $r(t)$ , is a quadratic function of the number of days,  $t$ , since the seed was sown. If the plant is not growing at seeding and ceases to grow after 40 days, and if the plant's maximum growth rate is a 8cm/day then  $r(t)$  is given by:

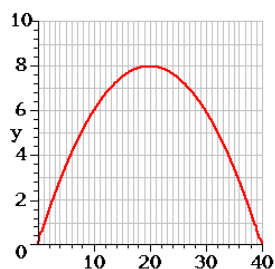
$$r(t) = -\frac{1}{50}(t - 20)^2 + 8 \quad (100\%)$$



Correct

Comment:

Here is a plot of the function  $r(t) = -\frac{1}{50}(t - 20)^2 + 8$ .





## Test 2: Functions – Composite and Inverse

### Question 1: Score 1/1

Your response

Given the following functions  $f(x) = 3x^2$ ,  $g(x) = x + 6$ ,  $h(x) = \frac{4}{x}$ , what are

1)  $f(g(x))$

$3 * (x+6)^2$  (50%)

2)  $f(g(h(x)))$

$3 * (4/x+6)^2$  (50%)



Correct

Comment:

### Question 2: Score 1/1

Your response

For the functions  $f(x) = 4x^3 + 6x + 4$  and  $g(x) = \sqrt{3x - 6}$  determine the function  $\frac{f}{g}$  and its domain.

Note: Use "infinity" for  $\infty$  and "sqrt(x)" for  $\sqrt{x}$ .

For the domain, enter your answer in interval notation. For example if  $0 \leq x < \infty$ , enter [0, infinity).

$\frac{f}{g} = (4x^3 + 6x + 4) / (3x - 6)^{(1/2)}$  (50%)

Domain =  $[6/3, \text{infinity})$  (50%)



Correct

Comment:

### Question 3: Score 1/1

Your response

For the functions  $f(x) = 5x^3$  and  $g(x) = \sqrt{x - 7}$  determine the function  $f - g$  and its domain.

Note: Use "infinity" for  $\infty$  and "sqrt(x)" for  $\sqrt{x}$ .

For the domain, enter your answer in interval notation. For example if  $0 \leq x < \infty$ , enter [0, infinity).

$f - g = 5x^3 - (x - 7)^{(1/2)}$  (50%)

Domain =  $[7, \text{infinity})$  (50%)



Correct

Comment:

#### Question 4: Score 1/1

Let  $f(x) = \frac{1}{5} (x^6 - 7)^{1/3}$ . Find the inverse function  $f^{-1}$ .



Correct

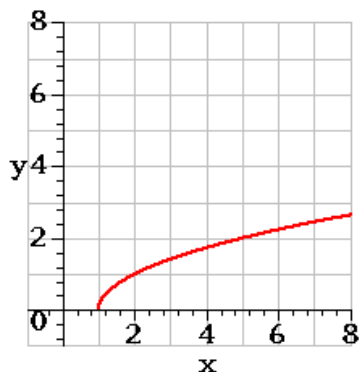
**Your Answer:**  $(125*x^3+7)^{(1/6)}$

**Comment:** To find the inverse, interchange  $x$  and  $y$  and solve for  $y$ .

#### Question 5: Score 1/1

Your response

For a given graph of the function  $f$



Correct

determine the

(i) domain of  $f = [1, \text{infinity})$  (25%)

range of  $f = [0, \text{infinity})$  (25%)

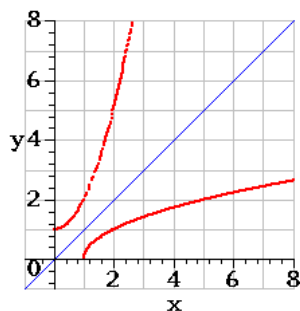
(ii) domain of  $f^{-1} = [0, \text{infinity})$  (25%)

range of  $f^{-1} = [1, \text{infinity})$  (25%)

Enter your answer in interval notation. For example if  $0 \leq x < \infty$ , enter  $[0, \text{infinity})$ .

**Comment:**

Here is the function, along with its inverse function



## Test 3: Trigonometric Functions

### Question 1: Score 1/1

Your response

Convert  $261^\circ$  to radian measure.

Give your answer to at least two decimal places.

Remember to enter 'Pi' for  $\pi$ .

4.56 (100%)



Correct

Comment:

### Question 2: Score 1/1

Your response

Convert the radian measure  $-\frac{11}{6}\pi$  to degrees. (Omit the degree symbol in your answer.)

-330 (100%)



Correct

Comment:

### Question 3: Score 1/1

Your response

Using special triangles, determine the following:

$\sin\left(\frac{\pi}{4}\right)$  1/2\*2^(1/2) (50%)

$\tan\left(\frac{\pi}{3}\right)$  3^(1/2) (50%)



Correct

Comment:

### Question 4: Score 1/1

Your response

The basic period of  $y = 3 \sin(6\pi x) + 11$  is

1/3 (100%)



Correct

Comment:

### Question 5: Score 1/1

Your response

Let  $f(x) = \cos(x)$  and  $g(x) = x^3 + x^2 + x$  then what is  $f(x) \circ g(x)$ ?  
 $\cos(x^3 + x^2 + x)$  (100%)



Correct

Comment:

### Question 6: Score 1/1

Your response

A quantity varies sinusoidally by  $y = K \sin(m(t - a)) + b$  where  $K$ ,  $m$ ,  $a$ , and  $b$  are constants. If the quantity has a period of 10 days, a maximum value of 180 and a minimum value of 80, what is the function that models this quantity?

Hint: Enter 'Pi' to represent  $\pi$ .

$y = 50 \sin(1/5 \pi t) + 130$  (100%)



Correct

Comment:

### Question 7: Score 1/1

Your response

Simplify the following using trig identities:  $4 \sec(x)^2 - 4 \tan(x)^2$   
4 (100%)



Correct

Comment:

### Question 8: Score 1/1

Your response

Using trig identities simplify the following:

$\cos(x) \tan(x) \cot(x) \csc(x)$   
 $\cos(x) / \sin(x)$  (100%)



Correct

Comment:

## Test 4: Logarithms and Exponents

### Question 1: Score 1/1

Your response

Solve for x in the following equation:

$$6 \cdot e^{6 \cdot x + 2} = 4$$

- .40 (100%)



Correct

Comment:

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### Question 2: Score 1/1

Your response

Which one of the following statements is false?

$$\frac{\log_3(3x+4)}{3} = 3x \quad (100\%)$$



Correct

Comment:

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### Question 3: Score 1/1

Your response

If  $2 \log_3(2) + \log_3(3x+1) = \log_3(x+3) + 2$  then x equals.

23/3 (100%)



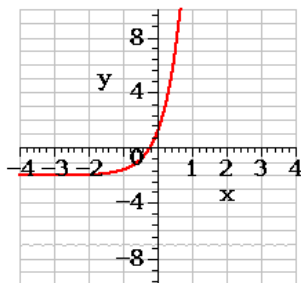
Correct

Comment:

#### Question 4: Score 1/1

Your response

State the equation of the function that is  $f(x) = 3e^{2x} - 2$  shifted to the left by 5 units and then shifted upwards by 3 units.



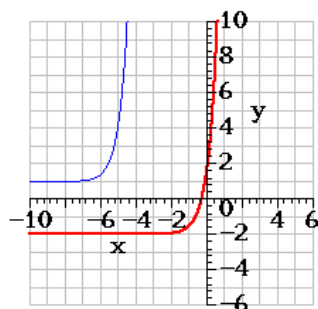
Correct

Note: Use exp(x) for  $e^x$ .

**3\*exp(2\*x+10)+1** (100%)

Comment:

Here is the two graphs plotted together.



#### Question 5: Score 1/1

Your response

Solve for  $x$  in the equation  $\ln(e^{6x+3}) = 7$ .

**2/3** (100%)



Correct

Comment:

#### Question 6: Score 1/1

Your response

Find the decay rate  $k$  of a radioactive substance if it has a half-life of 150 years.

Give your answer to at least 4 decimal places.

**0.0046** (100%)



Correct

Comment:

### Question 7: Score 1/1

#### Your response

A person take 150 mg of a drug. After 5 hours there is 30 percent of the drug left.

What is the decay rate of the drug?

Give your answer to at least two decimal places. You can omit units from your answer.



Correct

**.24** (100%)

## Test 5: Sequences and Series

### Question 1: Score 1/1

Your response

If the sequence  $\{X_n\}_{n=1}^{+\infty} = \{2, 5, 8, \dots\}$ , then  $X_{55} =$

164 (100%)



Correct

Comment:

Recall for an arithmetic sequence the general  $n$ 'th term is  $X_n = X_1 + (n - 1) \cdot d$

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### Question 2: Score 1/1

Your response

If the sequence  $\{X_n\}_{n=1}^{+\infty} = \{5, 20, 80, \dots\}$ , then  $X_6 =$

5120 (100%)



Correct

Comment:

Recall for an geometric sequence the general  $n$ 'th term is  $X_n = r^{n-1} \cdot X_1$

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### Question 3: Score 1/1

Your response

If the sequence  $\{X_n\}_{n=1}^{+\infty} = \{144, 36, 9, \dots\}$ , then  $X_5 =$

9/16 (100%)



Correct

Comment:

Recall for an geometric sequence the general  $n$ 'th term is  $X_n = r^{n-1} \cdot X_1$



## Question 4: Score 1/1

### Your response

Use the closed form formulas to evaluate the following series:

(i)  $\sum_{n=1}^{60} n = 1830$  (50%)

(ii)  $\sum_{n=1}^3 4 \cdot \left(\frac{1}{2}\right)^{n-1} = 7$  (50%)



Correct

### Comment:

Recall the following closed form formulas

$$\sum_{n=1}^N n = \frac{N \cdot (N + 1)}{2}$$

$$\sum_{n=1}^N g \cdot r^{n-1} = \frac{g \cdot (1 - r^N)}{1 - r}$$

## Question 5: Score 1/1

### Your response

Use the closed form formulas to evaluate the following series:

(i)  $\sum_{n=1}^{40} n = 820$  (33%)

(ii)  $\sum_{n=1}^{40} 30 = 1200$  (33%)

(iii)  $\sum_{n=1}^{10} (4^n + 1 - 4^n) = 4194300$  (33%)



Correct

For (iii) you do not have to simplify the exponents in the answer. (i.e., You could enter  $5^8$  instead of 390625).

### Comment:

Recall the following closed form formulas

$$\sum_{n=1}^N n = \frac{N \cdot (N + 1)}{2}$$

$$\sum_{n=1}^N k = N \cdot k$$

$$\sum_{n=M}^N (a_{n+1} - a_n) = a_{N+1} - a_M$$

## Question 6: Score 1/1

Your response

If  $\{f_n\}_{n=1}^{+\infty}$  is a Fibonacci sequence and  $f_6 = 42$  and  $f_8 = 110$  then  $f_1 =$

4 (100%)



Correct

Comment:

Recall that in a Fibonacci Sequence:  $f_1$  and  $f_2$  are specified and

$f_{n+2} = f_{n+1} + f_n$  for  $n = 1, 2, 3, \dots$

## Test 6: Limits and Continuity

### Question 1: Score 1/1

Your response

What is the following limit?

$$\lim_{x \rightarrow 2^-} x^2 + 3x + 4$$

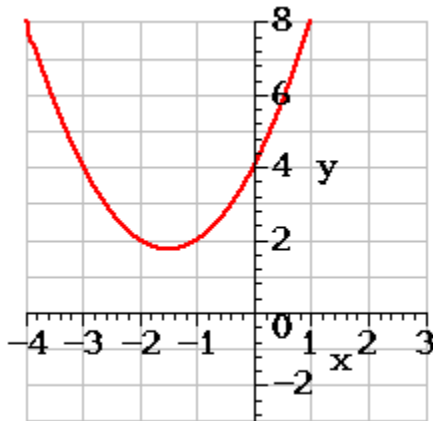
14 (100%)



Correct

Comment:

Here is what the step function looks like:



### Question 2: Score 1/1

Your response

What is the following limit?

$$\lim_{x \rightarrow 3} \frac{\sqrt{7} - \sqrt{10 - x}}{x - 3}$$

Note: For answers using  $\sqrt{x}$  use sqrt(x). For example, if your answer was  $\frac{2}{\sqrt{3}}$  you would enter 2/sqrt(3).

1/14\*7^(1/2) (100%)



Correct

Comment:

Remember, with a "0/0" limit, anything can happen!

### Question 3: Score 1/1

Your response

What is the following limit?

$$\lim_{x \rightarrow 4} \frac{x^2 - 6x + 8}{-4 + x}$$

2 (100%)



Correct

Comment:

Remember, with a "0/0" limit, anything can happen!

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### Question 4: Score 1/1

Your response

What is the following limit?

$$\lim_{x \rightarrow 2^+} \lfloor x + 3 \rfloor$$

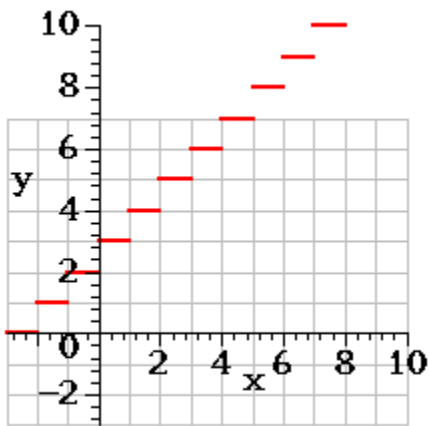
5 (100%)



Correct

Comment:

Here is what the function looks like:



## Test 7: Derivatives

### Question 1: Score 1/1

Your response

If  $f(x) = 7x^8$  then  $f'(x)$  is:

$8 \cdot 7^{\wedge}(x^{\wedge}8) \cdot x^{\wedge}7 \cdot \ln(7)$  (100%)



Correct

Comment:

Chain Rule: if  $f(x) = a^{g(x)}$  then  $f'(x) = a^{g(x)} \cdot g'(x) \cdot \ln(a)$

### Question 2: Score 1/1

Your response

If  $f(x) = x^{3/2} (4x - 6)$ , then  $f'(9)$ :

$243$  (100%)



Correct

Comment:

If a function is of the form  $f(x) = g(x) \cdot h(x)$  then the derivative is  $f'(x) = g'(x) \cdot h(x) + g(x) \cdot h'(x)$ .

### Question 3: Score 1/1

Your response

Using the power rule for derivatives, find the derivatives of the following functions.

i)  $y = 10x^2$

$20 \cdot x$  (50%)

ii)  $y = 2x^4$

$8 \cdot x^{\wedge}3$  (50%)



Correct

Comment:

The power rule says that the derivative of the function  $f(x) = c \cdot x^n$  is  $f'(x) = nc \cdot x^{n-1}$ .

#### Question 4: Score 1/1

Your response

Using the product rule, what is the derivative of the following function:  $f(x) = (6x^3 + 3x - 4)(5x^3 - 5x + 2)$



Correct

$(18x^2 + 3) \cdot (5x^3 - 5x + 2) + (6x^3 + 3x - 4) \cdot (15x^2 - 5)$  (100%)

Comment:

If a function is of the form  $f(x) = g(x) \cdot h(x)$  then the derivative is  $f'(x) = g'(x) \cdot h(x) + g(x) \cdot h'(x)$ .

#### Question 5: Score 1/1

Your response

Using the quotient rule, find the derivative of the following function:  $f(x) = \frac{7x^3 + x^2 + 6}{7x^3 + 4x + 1}$



Correct

$(21x^2 + 2x) / (7x^3 + 4x + 1) - (7x^3 + x^2 + 6) / (7x^3 + 4x + 1)^2 \cdot (21x^2 + 4)$  (100%)

Comment:

$$f'(x) = \frac{g'(x) \cdot h(x) - g(x) \cdot h'(x)}{[h(x)]^2}$$

If a function is of the form  $f(x) = \frac{g(x)}{h(x)}$  then the derivative is

.

#### Question 6: Score 1/1

Your response

Using the chain rule, find the derivative of the following function:  $y = (x^3 + 5x^2 + 2)^{10}$



Correct

$10 \cdot (x^3 + 5x^2 + 2)^9 \cdot (3x^2 + 10x)$  (100%)

Comment:

If a function is of the form  $f(x) = (g(x))^n$  then the derivative is  $f'(x) = n \cdot (g(x))^{n-1} \cdot g'(x)$ .

### Question 7: Score 1/1

Your response

Find the equation of the tangent line to  $f(x) = 15 \ln(x)$  at the point  $(5, 15 \ln(5))$ ?

$y = 3 \cdot x + 15 \cdot \ln(5) - 5$  (100%)



Correct

Comment:

### Question 8: Score 1/1

Your response

If  $y = e^{-\sin(x)}$  then  $\frac{d^2y}{dx^2}$  equals

$\sin(x) \cdot \exp(-\sin(x)) + \cos(x)^2 \cdot \exp(-\sin(x))$  (100%)



Correct

Comment:

### Question 9: Score 1/1

Your response

If  $f(x) = (-3x - 2)^{7/2}$  then  $f''(x)$  (the **second** derivative of  $f$  with respect to  $x$ ) is

$315/4 \cdot (-3x - 2)^{(3/2)}$  (100%)



Correct

Comment:

Chain Rule: if  $f(x) = (g(x))^n$  then  $f'(x) = n \cdot (g(x))^{n-1} \cdot g'(x)$

### Question 10: Score 1/1

Your response

If  $f(x) = 4x^3 e^{7x}$  then  $f''(x)$  (the **second** derivative of  $f$  with respect to  $x$ ) is

$24 \cdot x \cdot \exp(7 \cdot x) + 168 \cdot x^2 \cdot \exp(7 \cdot x) + 196 \cdot x^3 \cdot \exp(7 \cdot x)$  (100%)



Correct

Comment:

This question uses a combination of Chain Rule and Product Rule.

### Question 11: Score 1/1

Your response

If  $f(x) = 3 \sin(2x)$  then  $f'''(x)$  (the **third** derivative of  $f$  with respect to  $x$ ) is:

$-24 \cos(2x)$  (100%)



Correct

Comment:

Chain Rule: if  $f(x) = (g(x))^n$  then  $f'(x) = n \cdot (g(x))^{n-1} \cdot g'(x)$

### Question 12: Score 1/1

Your response

Use implicit differentiation to derive the following:

$$6x^3 + xy^2 + 6y = y^3$$

$(18x^2 + y^2) / (3y^2 - 2xy - 6)$  (100%)



Correct

Comment:

### Question 13: Score 1/1

Your response

The size of a cell culture at time  $t$  is given by the function  $P(t) = 2t + \frac{2}{t}$ ,  $t > 0.1$ .

Find the function that gives the instantaneous rate of change of the size of the cell culture for time  $t > 0.1$ .

$2 - 2/t^2$  (100%)



Correct

Comment:

To find the instantaneous rate of change, take the derivative.

### Question 14: Score 1/1

Your response

Find the derivative of the following function:  $y = (3x^3 + 5x^2 - 8)(10x^3 + 5)^8$

$(9x^2 + 10x) \cdot (10x^3 + 5)^8 + 240 \cdot (3x^3 + 5x^2 - 8) \cdot (10x^3 + 5)^7 \cdot x^2$  (100%)



Correct

Comment:

Chain Rule:  $f(x) = (g(x))^n$  then the derivative is  $f'(x) = n \cdot (g(x))^{n-1} \cdot g'(x)$ .

Product Rule:  $f(x) = h(x) \cdot g(x)$  then the derivative is  $f'(x) = h'(x) \cdot g(x) + h(x) \cdot g'(x)$ .



### Question 15: Score 1/1

Your response

If  $y = \cos(x)^{e^{6x}}$  then  $\frac{dy}{dx}$  equals



Correct

$\cos(x)^{\exp(6x)} * (6 * \exp(6x) * \ln(\cos(x)) - \exp(6x) * \sin(x) / \cos(x))$  (100%)

Comment:

If  $y = \cos(x)^{e^{6x}}$ , we use logarithmic differentiation to find  $y'$ .  
Start by taking the  $\ln$  of both sides.

## Test 8: Curve Sketching

### Question 1: Score 1/1

Your response

State the interval(s) for which the function  $y = \frac{1}{3}x^3 + \frac{5}{2}x^2 + 6x + 4$  is increasing?

Enter your answers using interval notation.

Use 'U' for union and 'infinity' for infinity.

Note: Increasing is not equivalent to  $f' > 0$ .

For instance,  $x^2$  has positive derivative on  $(0, \infty)$  but is increasing on  $[0, \infty)$ .

**$(-\infty, -3] \cup [-2, \infty)$**  (100%)



Correct

Comment:

### Question 2: Score 1/1

Your response

State interval(s) for which the function  $y = 2x^2 e^x$  is decreasing?

Enter your answers using interval notation.

Use 'U' for union and 'infinity' for infinity.

Note: Decreasing is not equivalent to  $f' < 0$ .

For instance,  $x^2$  has negative derivative on  $(-\infty, 0)$  but is decreasing on  $(-\infty, 0]$ .

**$[-2, 0]$**  (100%)



Correct

Comment:

### Question 3: Score 1/1

Your response

The critical numbers of the function  $f(x) = x\sqrt{6-x}$  are?

If you find just one critical number, leave the second box blank.

**6** (50%)

**4** (50%)



Correct

Comment:

The derivative of the function is zero at  $x = 4$ , hence 4 is a critical number.

The function has an endpoint at  $x = 6$ , hence 6 is a critical number.

#### Question 4: Score 1/1

Your response

The critical numbers of the function  $f(x) = 7x^3 - 5x^2 + 10$  are?

If you find just one critical number, leave the second box blank.

10/21 (50%)

0 (50%)



Correct

Comment:

The derivative of this function is zero at  $x = 10/21$  and  $x = 0$ .

---

#### Question 5: Score 1/1

Your response

Find all local extrema for the function  $f(x) = -x^3 + 4x^2$ .

local max at  $x = 8/3$  (50%)

local min at  $x = 0$  (50%)



Correct

Comment:

---

#### Question 6: Score 1/1

Your response

The point(s) of inflection for the curve  $f(x) = -3x^5 + 2x^4 - 7x$  occur at which  $x$ -values.

If you find more than one  $x$ -value, enter them separated by commas.

2/5 (100%)



Correct

Comment:

### Question 7: Score 1/1

Your response

The  $x$ -values of the inflection points for the function  $f(x) = -12x^4 + 10x^3 - 7$  are?

If you find just one inflection point, leave the second box blank.

5/12 (50%)

0 (50%)



Correct

#### Comment:

Inflection points occur where the second derivative changes sign. For this function, this occurs at  $x = 5/12$  and at  $x = 0$ .

---

### Question 8: Score 1/1

Your response

Assume the amount of rainfall  $R(t)$  (in cm) in Guelph during the 6 month rainy season is given by  $R(t) =$

$4t^2 - t^3$  for  $0 \leq t \leq 6$ , ( $t$  in months)

When is the amount of rainfall the greatest during the 6 month rainy season?

Do not round your answer (i.e., If you find  $t = 4/3$ , enter  $4/3$  not 1 or 2).

8/3 (100%)



Correct

#### Comment:

## Test 9: Differentials, Linear Approximation and

### Taylor Polynomials

#### Question 1: Score 1/1

Your response

Find the differential  $df(x)$  for the following function:

Note that the required 'dx', at the end of the answers is written in for you after the input box. **Omit dx from your answer.**

Note 2:  $e^x$  can be represented by exp(x). For example,  $4 \cdot e^{5x+2}$  would be entered as 4\*exp(5\*x+2).

1)  $\sqrt{4x-1}$

$2/(4x-1)^{(1/2)}$  (50%) dx

2)  $\frac{300}{\sqrt{x}}$

$-150/x^{(3/2)}$  (50%) dx



Correct

Comment:

Recall that the differential is defined by  $df(x) = f'(x) dx$

#### Question 2: Score 1/1

Your response

If the length of the sides of a cube changes from  $l = 5 \text{ cm}$  to  $5.004 \text{ cm}$ , then use differentials to determine the approximate change in (a) the volume and (b) the surface area of the cube in  $\text{cm}^3$ .

Note: The units are given for you after the input boxes. **Omit units from your answer.**

Give your answers to at least 2 decimal places.

(a) Approx. change in volume =  $.3$  (50%)  $\text{cm}^3$

(b) Approx. change in volume =  $.24$  (50%)  $\text{cm}^2$



Correct

Comment:

#### Question 3: Score 1/1

Your response

Use differentials to approximate the value of  $\sqrt[4]{17}$ .

Round your answer to 2 decimal places.

$65/64 * 16^{(1/4)}$  (100%)



Correct

Comment:

Recall that  $f(x) \approx f(x_0) + f'(x_0)(x - x_0)$ .

#### Question 4: Score 1/1

##### Your response

Use differentials to approximate the value of  $\sqrt{24}$ .

Give your answer to at least 2 decimal places.

$49/50*25^{(1/2)}$  (100%)



Correct

##### Comment:

Recall that  $f(x) \approx f(x_0) + f'(x_0)(x - x_0)$ .

#### Question 5: Score 1/1

##### Your response

If the radius of a sphere changes from  $r = 4$  to  $r = 4.001$  cm. Use differentials to determine the approximate change in (a) the volume and (b) the surface area.

Note 1: The units have been entered for you after the input box. **Omit units from your answer.**

Note 2: Use 3.14 for Pi. Give your answer to at least two decimal places.

(a)  $.20096$  (50%)  $\text{cm}^3$

(b)  $0.10048$  (50%)  $\text{cm}^2$



Correct

##### Comment:

#### Question 6: Score 1/1

##### Your response

Which of the following represents the linear approximation of  $f$  at  $x_0$ .

$f(x) \approx f(x_0) + f'(x_0)(x - x_0)$  (100%)



Correct

##### Comment:

Note that this is the tangent line to the graph of  $f$  at  $(x_0, f(x_0))$ .

#### Question 7: Score 1/1

##### Your response

The third order Taylor polynomial approximation about  $x_0 = 4$  of the function  $f(x) = e^{2x-8}$  is:

$-7+2*x+2*(x-4)^2+4/3*(x-4)^3$  (100%)



Correct

##### Comment:

### Question 8: Score 1/1

Your response

The third order Taylor polynomial approximation about  $x_0 = 0$  of the function  $\ln(x + 1)$  is:

$x - \frac{1}{2}x^2 + \frac{1}{3}x^3$  (100%)



Correct

Comment:

## Test 10: Difference Equations

### Question 1: Score 1/1

Your response

If  $X_{n+2} = X_{n+1} + 4 \cdot X_n$ , where  $X_0 = 2$  and  $X_1 = 7$  then  $X_5$  equals

275 (100%)



Correct

Comment:

### Question 2: Score 1/1

Your response

Determine the general solutions of the first order difference equations and the term  $X_{20}$ .

$$\Delta X_n = 5 \text{ and } X_0 = 300$$

400 (100%)



Correct

Comment:

Recall: if the Model is  $X_{n+1} = X_n + b$  then the general solution is  $X_n = X_0 + nb$

### Question 3: Score 1/1

Your response

Find the equilibrium values for the following difference equation:

$$X_{n+1} = \frac{1}{4}X_n(5 - X_n)$$

0 (50%)

1 (50%)



Correct

Comment:



#### Question 4: Score 1/1

Your response

If  $X_n = 2500 + 100 \cdot 4^n$  then determine the steady state  $\bar{X}$ .

Note: Enter "inf" for infinity and "D.N.E." for "Does Not Exist".

inf (100%)



Correct

Comment:

Recall: if the model is  $X_{n+1} = aX_n + b$  then the general solution is  $X_n = \frac{b}{1-a} + \left(X_0 - \frac{b}{1-a}\right)a^n$ . Then the steady state  $\bar{X}$  equals

$$\bar{X} = \lim_{n \rightarrow +\infty} X_n = \lim_{n \rightarrow +\infty} \left( \frac{b}{1-a} + \left(X_0 - \frac{b}{1-a}\right)a^n \right)$$

#### Question 5: Score 1/1

Your response

In an insecticide testing experiment on controlling the population of insects it was observed that each month there was a 15% increase in the insect population. After this increase, the insecticide was applied resulting in a 30% decrease in the population. After the decrease 70 insects were removed for testing.

State the difference equation representing the change in size of the insect population at the end of each month.

Note: Use "x" to represent  $x_n$ .

.805\*x-70 (100%)



Correct

Comment:

## Test 11: Log-Log Graphs

### Question 1: Score 1/1

Your response

If  $y = 400 \cdot x^{\frac{2}{3}}$  and  $y = 3600$ . Find a value of  $x > 0$  that makes this true.

27 (100%)



Correct

Comment:

### Question 2: Score 1/1

Your response

What is the slope of the line that is obtained by plotting the x, y values of the allometric function  $y = 128 \cdot x^{0.65}$  on log-log graph paper?

.65 (100%)



Correct

Comment:

### Question 3: Score 1/1

Your response

What is the y-intercept of the line that is obtained by plotting the x, y values of the allometric function  $y = 186 \cdot x^{0.08}$  on log-log graph paper?

Give your answer to 2 decimal places.

2.26951294 (100%)



Correct

Comment:

### Question 4: Score 1/1

Your response

What is the y-intercept of the line that is obtained by plotting the x, y values of the allometric function  $y = 26 \cdot (2^{0.5 \cdot x})$  on semi-log graph paper?

Give your answer to at least 2 decimals.

1.41 (100%)



Correct

Comment:

### Question 5: Score 1/1

Your response

What is the y-intercept of the line that is obtained by plotting the x, y values of the allometric function  $y = 90 \cdot (5^{0.17 \cdot x})$  on semi-log graph paper?

Give your answer to at least 2 decimal places.

**.12** (100%)



**Correct**

**Comment:**

## Test 12: Integration

### Question 1: Score 1/1

Your response

$$\int 2 \cdot x^4 dx$$

$2/5 \cdot x^5 + C$  (100%)



Correct

Comment:

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### Question 2: Score 1/1

Your response

$$\int x(2 - x^2)^3 dx$$

$-1/8 \cdot (2 - x^2)^4 + C$  (100%)



Correct

Comment:

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### Question 3: Score 1/1

Your response

$$\int \tan(x)^2 \sec(x)^2 dx$$

$1/3 \cdot \sin(x)^3 / \cos(x)^3 + C$  (100%)



Correct

Comment:

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### Question 4: Score 1/1

Your response

$$\int 3x^2 \csc(x^3) \cot(x^3) dx$$

$-\csc(x^3) + C$  (100%)



Correct

Comment:

### Question 5: Score 1/1

Your response

$$\int \frac{6x^5 + 14x}{x^6 + 7x^2 + 3} dx$$

$\ln(x^6 + 7x^2 + 3) + C$  (100%)



Correct

Comment:

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### Question 6: Score 1/1

Your response

Assume that a tumor grows at rate  $G(t) = \frac{4}{3}t^{1/3}$  mg/day at time  $t$  days, and at  $t = 0$  the size of the tumor is 5 mg. The size of the tumor,  $S(t)$ , at time  $t$  days is:

$t^{4/3} + 5$  (100%)



Correct

Comment:

## Test 13: Definite Integrals

### Question 1: Score 1/1

Your response

Evaluate the following definite integral.

$$\int_1^3 7 \cdot x^3 dx$$

140 (100%)



Correct

Comment:

### Question 2: Score 1/1

Your response

Solve the following definite integral.

$$\int_1^2 x \cdot (4 - x^2)^3 dx$$

81/8 (100%)



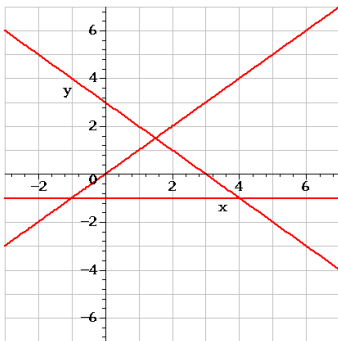
Correct

Comment:

### Question 3: Score 1/1

Your response

Determine the area of the region bounded between the three graphs  $f(x) = -1$ ,  $g(x) = -x + 3$ , and  $h(x) = x$



Correct

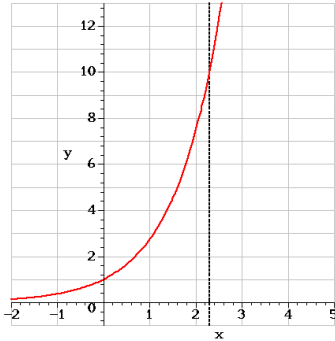
25/4 (100%)

Comment:

#### Question 4: Score 1/1

Your response

Determine the area of the finite region bounded by the graphs of  $f(x) = e^x$ , the x-axis, the y-axis, and the line  $x = \ln(10)$



Correct

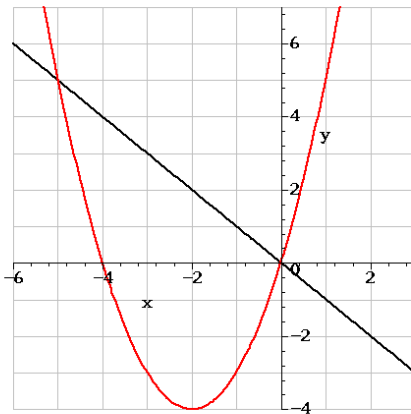
9 (100%)

Comment:

#### Question 5: Score 1/1

Your response

Determine the area of the region bounded between the two graphs  $f(x) = x^2 + 4x$  and  $f(x) = -x$ .



Correct

125/6 (100%)

Comment:

### Question 6: Score 1/1

Your response

Assume that a tumor grows at rate of  $G(t) = \frac{8}{3} \sqrt[3]{t}$  mg/day at time  $t$  days. What is the total change in

the mass of the tumor from  $t = 0$  to  $t = 8$  days.

Round your answer to two decimal places.

30.00 (100%)



Correct

Comment:

### Question 7: Score 1/1

Your response

Assume that the rate of excretion for a drug is given by  $R(t) = \frac{10}{7} t^{4/3}$  mg/hour. What is the total

amount of the drug excreted from  $t = 0$  to  $t = 5$  hours.

Round your answer to two decimal places.

77.76 (100%)



Correct

Comment:

### Question 8: Score 1/1

Your response

Assume that a tumor grows at rate of  $G(t) = \frac{1}{4} t^{\frac{3}{5}}$  mg/day at time  $t$  days. What is the average of  $G(t)$  in

the first 32 days?

Round your answer to 2 decimal places.

1.25 (100%)



Correct

Comment: